Logic for Computer Science.
Knowledge Representation and Reasoning.

Lecture Notes

for

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Other support material:

http://home.agh.edu.pl/~ligeza

https://ai.ia.agh.edu.pl/pl:dydaktyka:logic:start

Logics - what is that?

Definition 1 *Logika (logos — rozum, słowo, myśl) — nauka o sposobach jasnego i ścisłego formułowania myśli, o regułach poprawnego rozumowania i uzasadniania twierdzeń.*

Definition 2 *Logic (from the Ancient Greek) is the systematic study of the forms of inference, the relations that lead to the acceptance of one proposition, the conclusion, on the basis of a set of other propositions, the premises. More broadly, logic is the analysis and appraisal of arguments.*

Logic = (Axioms) + (Formal Models) + (Valid Inference Rules)

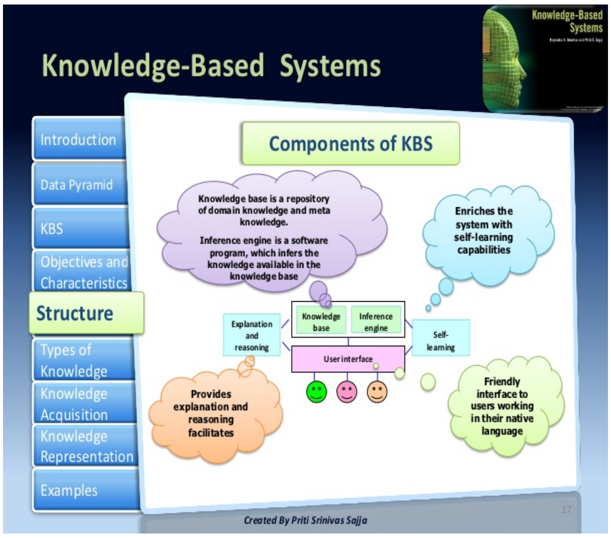
Tools:

- Formal Models – built with Formal Languages:
  - alphabet,
  - syntax,
  - semantics,
  - axioms,
  - inference rules,
  - inference strategies (methods of Theorem Proving);

Supplementary Tools

- Visualization — e.g. Venn's Diagrams,
- Tables (complete specification of cases),
- Trees (better visual presentation),
- Diagrams (graphs; AND-OR graphs; schemes),
- Software tools (e.g. SAT solvers, Prolog, ASP, ATP).

Knowledge-Based Systems: Basic Concepts



We need some tools:

- A formal language for KRR:
  - alphabet,
  - syntax,
  - semantics,
  - inference rules;
- various goals — various types of reasoning,
- inference strategies,
- knowledge acquisition,
- knowledge verification,
- minimal knowledge representation (uniform representation),
- internal knowledge structure (knowledge graphs),
- user interface, explanations, learning, adaptation, optimization.

Przykłady... wieloznaczność języka naturalnego

```
Żona do męża informatyka
- Kup parówki, a jak będą jajka -- to kup 10.
Mąż w sklepie:
- Są jajka?
- Są.
- To poproszę 10 parówek.
```

Przykłady wyrażeń w języku naturalnym:

- *Mądrej głowie dość po słowie* (Wise head - a word is enough).
- *Mądrej głowy włos się nie trzyma*.
- *Dobry kogut to chudy kogut* (A good cock is a skinny cock).
- *Iloma językami władasz, tylekroć jesteś człowiekiem*
  (How many languages do you speak, so many times you are human; "Wie viele Sprachen du sprichst, sooft mal bist du Mensch.").
  Logika to także język.
- *Historia uczy jednego: nigdy, nikogo, niczego nie nauczyła*.
  Czy to zdanie może być prawdziwe?

Logic – a tool for Knowledge Representation and Reasoning (KRR)

Observation:

Not all the systems/processes/... can be modelled with numerical tools.

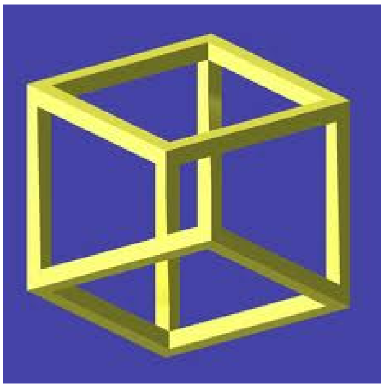
Simultaneously, Natural Language (NL):

- is often inadequate/too poor/...,
- imprecise, incomplete,
- ambiguous– there are different interpretations possible,
- hard for automated processing,
- can lead to inconsistent models/theories,
- is redundant,
- requires the knowledge of *domain ontology*.

Formal language – selected requirements:

- adequate,
- precise,
- unambiguous,
- automatic processing,
- should assure (consistency),
- should allow to produce validit/sound conclusions,
- can assure completeness,
- should be not redundant.

Something has gone wrong...



What one needs logic for?

Po co komu krawat? *Sprzedawca krawatów...*

- logic allows to formulate definitions of concepts (marriage, fast tramway, *średnioroczny wzrost cen, jakość wykształcenia, jakość kształcenia, jakość procesu kształcenia,...*),
- and the relationship among them — np. taxonomies, mutual relations/roles, ontologies,
- logic introduces order into discussion; rational discussion based on *argumentation*,
- logic provides formal Knowledge Representation,
- logic provides valid/correct inference methods — deduction,
- but also: induction and abduction,
- logically specified knowledge can undergo formal analysis:
  - internal; consistency (wewnętrzna niesprzeczność/spójność)),
  - completeness (zupełność),
  - minimal representation (minimalna reprezentacja),
  - logical consequence (wynikanie logiczne — logiczna konsekwencja),
  - satisfiability or unsatisfiability (spełnialność lub niespełnialność).
- logic is a formal tool (is widely applied in various domains...).

Where logic is not applied/applicable?? What does not undergo logical rules of reasoning?

Some hot areas of application

- Computer Programming,
- Knowledge-Based Systems (KBS),
- Automated Theorem Proving (ATP),
- Logic Programming,
- SAT,
- Constraint Programming,
- Answer Set Programming; ASP,
- Systems Design and Analysis,
- Model Checking,
- systems/software certification.

Logic – OK – but still some problems...

Natural language:

Paradoks kłamcy:

- *I always lie!* (Eubulides),
- *The Cretans always lie* (Epimenides; he was a Cretan himself),
- Page 1: *The sentence on the other side is true*; Page 2: *The sentence on the other side is false*.
- **Mukator** — Co to jest mukator? To jest coś, co *nie daje się zdefiniować*.

Formal language:

Matematyka:

- Paradox of the set of all sets (Cantor, 1899),
- Russel's paradox (1901): consider $V = \{X : X \notin X\}$. Does it hold that: $V \in V$?
- Barber Paradox: *A barber shaves all residents of the city who do not shave themselves; what should he do with himself?*

Examples of paradoxes: `http://pl.wikipedia.org/wiki/Paradoks`;
`https://en.wikipedia.org/wiki/Paradox`

Formal Knowledge Representation Language: Specification of Requirements

Knowledge Representation Language:

- unique interpretation,
- expressive power and precision, but
- automated processing,
- adequate for the domain,
- readable for man and machine,
- extensible,
- ...

Knowledge Base:

- consistent, (internal consistency)
- complete,
- valid; sound, (external consistency)
- non-redundant,
- efficient in problem solving, (optimal)
- ...

Logic — how it works?

Logic – formal language:

- alphabet,
- *syntax*,
- *semantics*,
- axiomatization

$$\models p \vee \neg p$$

$$\not\models p \wedge \neg p$$

- equivalency-preserving transformations,
- inference rules (logical consequence),
- inference chain – derivation,
- problem, hypothesis, question,
- answer, solution, proof.

System modeling :

- language selection – for adequate modeling,
- model building (inputs – outputs – internal structure),
- model analysis (verification, validation),
- model exploration – theorem proving, SAT,
- model tuning and adaptation; learning.

Example: a simple intro to Propositional Calculus

Alphabet:

- P — set of propositional symbols,
- logical connectives: $\neg$, $\wedge$, $\vee$, $\Rightarrow$,
- parentheses.

Syntax:

- every $p \in P$ is a formula (atomic formula; positive literal),
- if ϕ, ψ are formulas, then:
  - $\neg(\phi)$ is a formula (also: $\neg(\psi)$),
  - $(\phi) \wedge (\psi)$ is a formula,
  - $(\phi) \vee (\psi)$ is a formula,
  - $(\phi) \Rightarrow (\psi)$ is a formula;
  - *no other expression is a formula*.

Each well-formed formula (wff) has a uniquely defined tree structure.

Semantics:

Interpretation — any function I of the form:

$$I\colon P \rightarrow \{\mathbf{T}, \mathbf{F}\}$$

The definition of interpretation is extended over the set of all formulae (how to do that ???).

Notation: $I(\phi) = \mathbf{T}$ we shall write $\models_I \phi$; $I(\phi) = \mathbf{F}$ we shall write $\not\models_I \phi$.

For any wff a truth table can be built.

The symbols $\mathbf{T}$ ($True, true$) and $\mathbf{F}$ ($False, false$) are often replaced with $\mathbf{1}$ and $\mathbf{0}$, respectively.

Rules of inference

https://en.wikipedia.org/wiki/List_of_rules_of_inference: examples + example derivations

Example rules of inference (derivation rules):

Detachment Rule (*Modus Ponens*; Reguła odrywania):

$$\frac{\phi, \phi \Rightarrow \psi}{\psi}$$

Contraposition (*Modus Tollens*; Wnioskowanie przez zaprzeczenie):

$$\frac{\phi \Rightarrow \psi, \neg\psi}{\neg\phi}$$

Disjunctive Syllogism; Disjunction Elimination by Contradiction (Reguła potwierdzania przez zaprzeczenie (wykluczenie)):

$$\frac{\neg\phi, \phi \vee \psi}{\psi}$$

Transitivity Rule (Reguła przechodniości):

$$\frac{\phi \Rightarrow \varphi, \varphi \Rightarrow \psi}{\phi \Rightarrow \psi}$$

Resolution Rule (Reguła rezolucji):

$$\frac{\phi \vee \neg p, p \vee \psi}{\phi \vee \psi}$$

What about the following rules? !!!  !!!

Implication + positive conclusion?

$$\frac{\phi \Rightarrow \psi, \psi}{\phi}$$

Implication + negative premise

$$\frac{\neg\phi, \phi \Rightarrow \psi}{\neg\psi}$$

2 Implications + conjunction

$$\frac{p \wedge q, p \Rightarrow r, q \Rightarrow s}{r \wedge s}$$

Proof by Cases: 2 Implications + disjunction

$$\frac{p \vee q, p \Rightarrow r, q \Rightarrow r}{r}$$

Constructive Dilemma: 2 Implications + disjunction

$$\frac{p \vee q, p \Rightarrow r, q \Rightarrow s}{r \vee s}$$

Dual Resolution Rule (Reguła rezolucji dualnej):

$$\frac{\phi \wedge \neg p, p \wedge \psi}{\phi \wedge \psi}$$

Example — układ EX-OR

```
% Definicje działania bramek podstawowych
not(1,0).
not(0,1).

and(0,0,0).
and(0,1,0).
and(1,0,0).
and(1,1,1).

or(0,0,0).
or(0,1,1).
or(1,0,1).
or(1,1,1).

% Definicja przykładowego układu - xor

xor(Input1,Input2,Output) :-
not(Input1,N1),
not(Input2,N2),
and(Input1,N2,N3),
and(Input2,N1,N4),
or(N3,N4,Output).
```

An Example — układ EX-OR

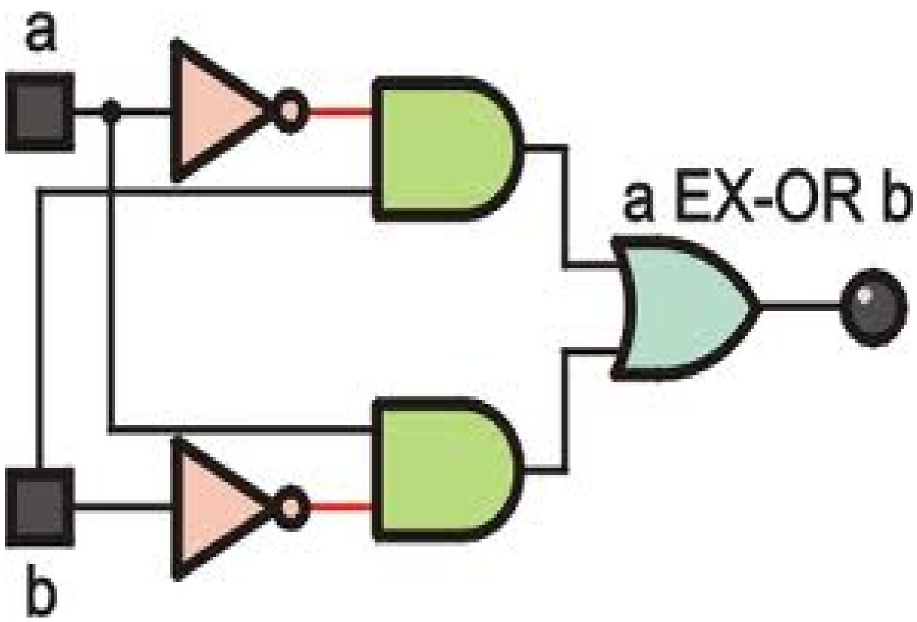
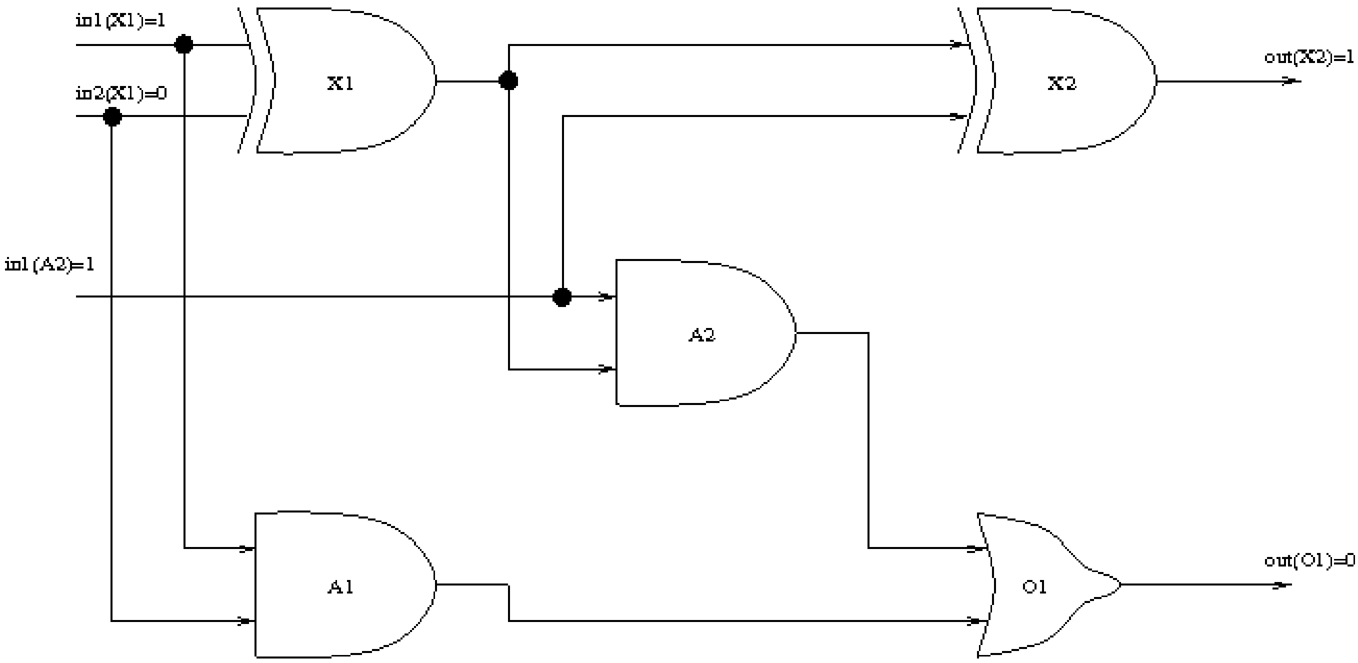


Figure 1: EX-OR digital circuit

A Practical Example

```
foo(I1,I2,I3,O1,O2) :-
   xor(I1,I2,X1),
   xor(X1,I3,O1),
   and(I1,I2,A1),
   and(X1,I3,A2),
   or(A1,A2,O2).
```



Logical inferences — examples

Examples of logical inference:

- deduction — fundamental logical inference (hypotheses proving);
- SAT: satisfiability checking — model search, constraint analysis;
- detection of inconsistencies; consistency-based reasoning (CBR);
- tautology verification;
- abduction — making hypotheses (finding out the causes);
- induction — generalization;
- case-based reasoning, reasoning by analogy, plausible reasoning; fuzzy reasoning, probabilistic reasoning,...

Examples – applications – and their classification:

- deductive theorem proving (deduction rules, Herbrand, Fitch, forward checking),
- formula for the sum of the infinite geometric sequence (deduction vs. induction),
- SEND + MORE = MONEY,
- the Einstein/Zebra problem,
- diagnosis of multiplier-adder,
- a promise of earnings above average for everyone,
- rational result of power of two irrational numbers,
- polynomial example ($P > 0$ and $\forall \epsilon > 0 \colon P < \epsilon$),

- practical examples:
  - 7-segment display - for 16-bar code
  - 3/5 or 4/7 voting system
  - lock-2-doors to the bank,
  - a 2-way pass-through switch,
  - railway crossing,

Logic for KRR – Tasks and Tools

- Theorem Proving – Verification of Logical Consequence:

$$\Delta \models H;$$

- Automated Inference – Derivation:

$$\Delta \vdash H;$$

- SAT (checking for models) – satisfiability:

$$\models_I H;$$

- un-SAT verification – unsatisfiability:

$$\not\models_I H \quad \text{for any interpretation I;}$$

- Tautology verification (completeness):

$$\models H$$

- valid inference rules – checking:

$$(\Delta \vdash H) \quad \longrightarrow \quad (\Delta \models H)$$

- complete inference rules – checking:

$$(\Delta \models H) \quad \longrightarrow \quad (\Delta \vdash H)$$

Inference example

A – signal from process,

P – signal added to a queue,

B – signal blocked by process,

D – signal received by process,

S – state of the process saved,

M – signal mask read,

H – signal management procedure activated,

N – procedure executed in normal mode,

R – process restart from context,

I – process must re-create context.

Rules — axiomatization:

$A \longrightarrow P$,

$P \wedge \neg B \longrightarrow D$,

$D \longrightarrow S \wedge M \wedge H$,

$H \wedge N \longrightarrow R$,

$H \wedge \neg R \longrightarrow I$,

Facts:

A, $\neg B$, $\neg R$. What can be inferred from the facts? What are the logical consequences of the KB? Which facts are True and which ones are False? Is the system complete?

A Problem to Consider: Tracking the Murderer

Some knowledge specification — in natural language:

- If Sarah was drunk then either James is the murderer or Sarah lies,
- Either James is the murderer or Sarah was not drunk and the crime took place after midnight,
- If the crime took place after midnight then either James is the murderer or Sarah lies,
- Sarah does not lie when sober.

Introduction symbols and transformation to formal specification:

- A = James is the murderer,
- B = Sarah is drunk,
- C = Sarah lies,
- D = The murder took place after midnight.

$$B \Longrightarrow A \vee C$$

$$A \vee (\neg B \wedge D)$$

$$D \Longrightarrow (A \vee C)$$

$$C \Longrightarrow B$$

Questions:

Who is the murderer? Which facts are true/false? Is the system consistent? How many models does it have (if consistent)? What are the exact models?

Example: Unicorn

Given the following Knowledge Base (KB):

- If the unicorn is mythical, then it is immortal
- If the unicorn is not mythical, then it is a mortal mammal
- If the unicorn is either immortal or a mammal, then it is horned
- The unicorn is magical if it is horned

answer the following questions:

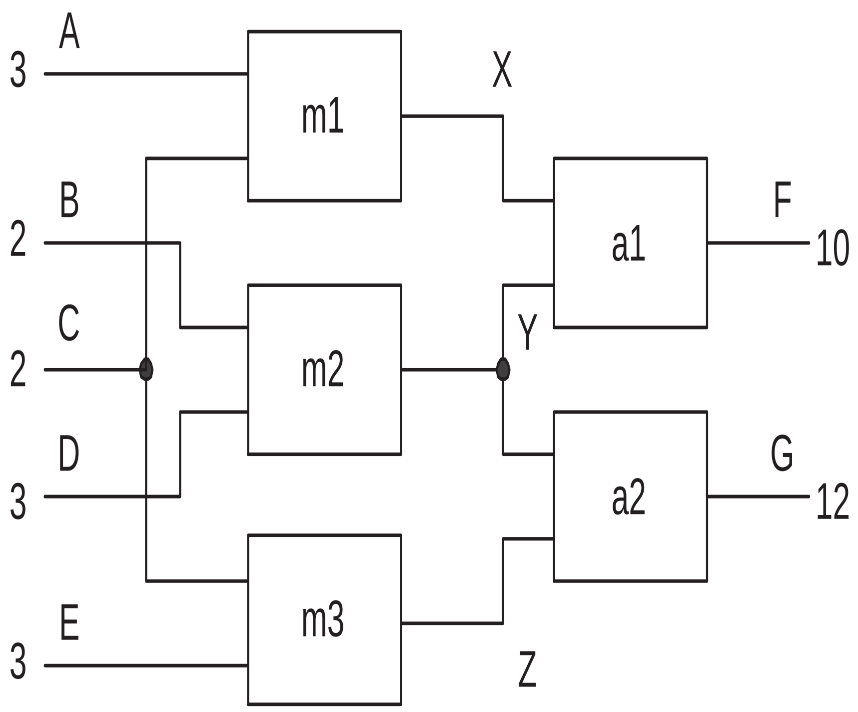
- Is the unicorn mythical? (M)
- Is it magical? (G)
- Is it horned? (H)

In terms of logic:

$$\text{KB} \models G, H, M$$

$$\text{KB} \vdash G, H, M$$

Abductive Inference and Consistency-Based Reasoning



Questions:

- Does the system work OK (Fault Detection)?
- Which component(s) is(are) faulty (Fault Isolation)?
- What are the diagnoses (minimal diagnoses)?