
# Essential Thinking. The Art of Creative Thinking for Problem Solving.

Lecture Notes

and

Support Stuff

Faculty EAIiIB-IEiT AGH

Antoni Ligęza

Other support material:

`http://home.agh.edu.pl/~ligeza`

`https://ai.ia.agh.edu.pl`

# A Cold Start: Selected Problems to Fertilize our Brains

Consider the following example problems to be analyzed and if you have time and enthusiasm – solved. But this is not obligatory; I provide these problems just to let you know the idea of examples I will use to illustrate the lectures. An to initialize some mental process in your minds...

Just have a look at these 10 example problems. If you know some of them – that is O.K. If you know how to solve them – it is even more than O.K. But please do not look into internet. Just try your imagination. And try to think and focus on the methodological issues provided below the 10 problems.

Selected 10 problems:

1. The Money Change Problem. Being given (unlimited) number of coins of several different types = positive integer values, say $m_1, m_2, \ldots, m_N$, where $N$ is the numer of types (e.g. 1 PLN, 2 PLN, 5 PLN; here $N$ = 3) how to find the numbers (positive integers or zero) $c_1, c_2, \ldots, c_N$ such that
$$S = \sum_{i=1}^{N} c_i m_i,$$
where S is a given positive integer? In other words, how to make the change of some $S$ with predefined coins? And perhaps so that the number of coins is minimal?

2. Prove that $\sqrt{2}$ is an irrational number.

3. There are:
- rational numbers: 1,2,3,...,1/2, 1/3, 3/5,...
- irrational numbers: $\pi$, $e$, $\sqrt{2}$, $\sqrt{3}$,...
Operations on irrational numbers can lead to rational numbers, e.g.
$$\sqrt{2} \cdot \sqrt{2} = 2$$

Problem: do there exists two irrational numbers, say $a$ and $b$, such that:

$$a^b$$

is a rational number?

The answer should be: YES or NO.

4. A Polynomial Problem. Does there exists a polynomial function $P$ satisfying the following conditions:

   - it is always strictly greater than zero:

     $$P > 0$$

   - for any (small) $\epsilon > 0$, there exists a value of the polynomial less then $\epsilon$

     $$\forall \epsilon > 0 \quad \exists P_\epsilon : \quad P_\epsilon < \epsilon$$

   - Prove or
   - Disprove.

5. Bridge and Torch. Four people come to a river in the night. There is a narrow bridge, but it can only hold two people at a time. They have one torch and, because it's night, the torch has to be used when crossing the bridge. Person A can cross the bridge in 1 minute, B in 2 minutes, C in 5 minutes, and D in 8 minutes. When two people cross the bridge together, they must move at the slower person's pace. The question is, can they all get across the bridge if the torch lasts only 15 minutes?

6. Missionaries and Cannibals. In the missionaries and cannibals problem, three missionaries and three cannibals must cross a river using a boat which can carry at most two people, under the constraint that, for both banks, if there are missionaries present on the bank, they cannot be outnumbered by cannibals (if they were, the cannibals would eat the

missionaries). The boat cannot cross the river by itself with no people on board. And, in some variations, one of the cannibals has only one arm and cannot row.

7. The Spoiled Chessboard Problem. There is a chessboard $8 \times 8$, and a set of domino tiles, such that each of them covers exactly two neighbouring fields. Let us cut of the opposite corner fields, say a1 and h8, being located on the opposite ends of a diagonal. Is it possible to cover the chessboard 1:1, so that (i) every field is covered and n(ii) none of the the dominoes are sticking out of the board?

8. Tower of Hanoi. The Tower of Hanoi (also called the Tower of Brahma or Lucas' Tower and sometimes pluralized as Towers) is a mathematical game or puzzle. It consists of three rods and a number of disks of different sizes, which can slide onto any rod. The puzzle starts with the disks in a neat stack in ascending order of size on one rod, the smallest at the top, thus making a conical shape. The objective of the puzzle is to move the entire stack to another rod, obeying the following simple rules:

   - Only one disk can be moved at a time.
   - Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod.
   - No larger disk may be placed on top of a smaller disk. How to perform this operation?

9. A Cryptoarithmetic Problem. Find the digits to replace the variables, so that:

```
  SEND
+ MORE
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```

```
    MONEY
```

Different letters denote different digits; the leading digits are non-zero.

10. The Einstein Problem. Einstein's riddle:

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1. There are 5 houses in five different colors.
2. In each house lives a person with a different
   nationality.
3. These five owners drink a certain type of beverage,
   smoke a certain brand of cigar and keep a certain pet.
4. No owners have the same pet, smoke the same brand of
   cigar or drink the same beverage.
The question is: Who owns the fish?
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Hints:

- the Brit lives in the red house
- the Swede keeps dogs as pets
- the Dane drinks tea
- the green house is on the left of the white house
- the green house's owner drinks coffee
- the person who smokes Pall Mall rears birds
- the owner of the yellow house smokes Dunhill
- the man living in the center house drinks milk
- the Norwegian lives in the first house
- the man who smokes blends lives next to the one who keeps cats
- the man who keeps horses lives next to the man who smokes Dunhill
- the owner who smokes BlueMaster drinks beer
- the German smokes Prince

- `the Norwegian lives next to the blue house`
- `the man who smokes blend has a neighbor who drinks water`

It is believed that Einstein wrote this riddle this century. He said that 98% of the world could not solve it.

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Methodological issues:

1. Can you say that the problem is well defined? Does it have a solution? Is the solution unique?
2. How to classify each particular problem in terms of its characteristic features and perhaps membership of some known class of problems?
3. If you well-classify the problem – is there an algorithmic procedure to solve it?
4. What Knowledge Representation Method would be most convenient?
5. What kind of Reasoning Methods would be most useful?
6. What domain-dependent specific rules/tools/methods can be applied?
7. What heuristic rules can be useful?
8. How to check that the solution is correct?
9. Can you generalize the result? What are the lessons learned?
10. Can you think of using some tools (e.g. computer programs, programming languages, pictures) to help you solve the problem?